Problem 1: (5pts)

$x_j = 1$ if the jth healthcare center is built and 0 otherwise / $j = 1, 2, 3$ and 4 1

$\text{Min}(Z) = \cancel{1000}\,[60x_1 + 70x_2 + 50x_3 + 40x_4]$ 1

$x_1 + x_2 \geq 1$ 0,5

$x_1 + x_2 + x_3 \geq 1$ 0,5

$x_1 + x_4 \geq 1$ 0,5

$x_2 + x_4 \geq 1$ 0,5

$x_3 + x_4 \geq 1$ 0,5

$x_1 + x_2 + x_3 + x_4 \leq 3$ 0,5

$x_j = 1$ or 0 ; $j = 1, 2, 3$ and 4.

Problem 2: (5pts)

$$\begin{bmatrix} 12 & 11 & 8 & 13 \\ 13 & 16 & 15 & 10 \\ 11 & 15 & 13 & 14 \\ 14 & 13 & 12 & 9 \end{bmatrix} \xRightarrow{\text{Row reduction}} \begin{bmatrix} 4 & 3 & 0 & 5 \\ 3 & 6 & 5 & 0 \\ 0 & 4 & 2 & 3 \\ 5 & 4 & 3 & 0 \end{bmatrix}$$ 1

Column reduction $\Rightarrow$
$$\begin{bmatrix} 4 & 0 & 0 & 5 \\ 3 & 3 & 5 & 0 \\ 0 & 1 & 2 & 3 \\ 5 & 1 & 3 & 0 \end{bmatrix}$$ 1

Making squares

$$\begin{bmatrix} 4 & \boxed{0} & \cancel{0} & 5 \\ 3 & 3 & 5 & \boxed{0} \\ \boxed{0} & 1 & 2 & 3 \\ 5 & 1 & 3 & \cancel{0} \end{bmatrix}$$ 1

$$\begin{bmatrix} 4 & \cancel{0} & \boxed{0} & 6 \\ 2 & 2 & 4 & \boxed{0} \\ \boxed{0} & 1 & 2 & 4 \\ 4 & \boxed{0} & 2 & \cancel{0} \end{bmatrix}$$ The solution is optimal. 1

operator 1 $\longrightarrow$ Machine 3
operator 2 $\longrightarrow$ Machine 4
operator 3 $\longrightarrow$ Machine 1
operator 4 $\longrightarrow$ Machine 2 1

$Z = 8 + 10 + 11 + 13 = 42$.

Problem 3: (10pts)

1- a competitive game ~~with~~ only two players where one player's gain is equal the other player's loss. 1

لعبة تنافسية بين لاعبين فقط حيث أرباح أحد اللاعبين هي نفسها خسائر اللاعب الآخر

2- [Maximin = -1] $\neq$ [Minimax = 2] 1

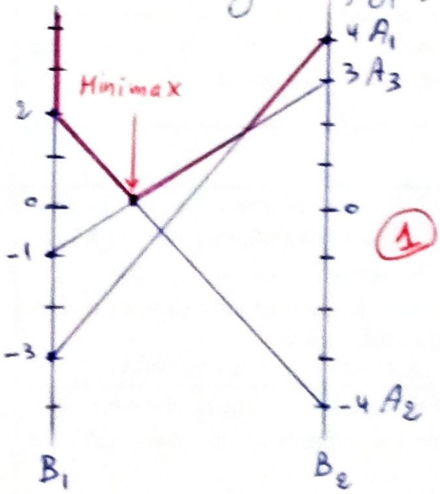
there is no saddle point, Thus The appropriate strategy is mixed strategy. 1

3- We note That The column (B_3) is greater Than The column (B_1), So The B_3 is removed.

Strategy	B_1	B_2
A_1	-3	4
A_2	2	-4
A_3	-1	3

1

here matrix is of $(M \times 2)$ Type, we solve it graphically



1

Strategy	B_1	B_2	
A_2	2	-4	P_1
A_3	-1	3	P_2
	q_1	q_2	

Player A 1

$$\left.\begin{aligned} 2P_1 - 1P_2 &\geq V \\ -4P_1 + 3P_2 &\geq V \\ P_1 + P_2 &= 1 \end{aligned}\right\} \Rightarrow P_1 = \frac{2}{5}\,;\ P_2 = \frac{3}{5} \text{ and } V = \frac{1}{5} \quad A\left(0; \frac{2}{5}; \frac{3}{5}\right)$$ 0,5

Player B 1

$$\left.\begin{aligned} 2q_1 - 4q_2 &\leq V \\ -q_1 + 3q_2 &\leq V \\ q_1 + q_2 &= 1 \end{aligned}\right\} \Rightarrow q_1 = \frac{7}{10}\,;\ q_2 = \frac{3}{10}\,;\ V = \frac{1}{5} \quad B\left(\frac{7}{10}; \frac{3}{10}, 0\right)$$ 0,5

4- $\text{Max}(Z) = V$

$-3P_1 + 2P_2 - P_3 \geq V$

$4P_1 - 4P_2 + 3P_3 \geq V$ 2

$P_1 + P_2 + P_3 = 1$

$P_1, P_2, P_3 \geq 0$ and V is free